

ECE 260 course notes

Every module's technical content in one place, by topic: the power system, AC circuits and three-phase power, magnetic circuits, transformers, DC machines, synchronous machines and induction motors – formulas, the highest-value worked examples, and practice with answers.

The power system

Generation at 13.8–24 kV, transmission meshed for reliability at 230–765 kV, sub-transmission at 69–138 kV, distribution radial for cost at 2.2–46 kV down to a service voltage of 120–480 V.

Phasors, impedance, and instantaneous power

Module 2 part A: the circuit language every later module is written in.

A lowercase symbol is instantaneous, a capital is RMS.

KEY RELATION

$$v(t) = \sqrt{2} V \cos \omega t, \quad i(t) = \sqrt{2} I \cos(\omega t - \phi), \quad V = \frac{V_{max}}{\sqrt{2}}; \quad \bar{V} = V \angle 0^\circ, \quad \bar{I} = I e^{-j\phi} = I \angle -\phi.$$

A lagging current has a negative angle: $\phi > 0$ puts $\bar{I}$ below the real axis.

KEY RELATION

$$Z = \frac{V \angle \theta_v}{I \angle \theta_i} = |Z| \angle \theta, \quad j\omega L = jX_L, \quad -j\frac{1}{\omega C} = -jX_C, \quad Z_{RLC} = R + j(X_L - X_C).$$

Instantaneous power splits into a part that never goes negative and a part that averages to zero:

KEY RELATION

$$p(t) = P(1 + \cos 2\omega t) + Q \sin 2\omega t, \quad P = VI \cos \phi \text{ [W]}, \quad Q = VI \sin \phi \text{ [VAR]}.$$

P is never negative; Q averages exactly zero – a pure L or C borrows energy and returns it each cycle.

KEY RELATION

$$PF = \cos \phi, \quad RF = \sin \phi, \quad \phi > 0 \text{ for lagging current.}$$

Lagging is inductive, leading capacitive; always quote which.

Complex power and power-factor correction

Module 2 part B.

KEY RELATION

$$S = \bar{V}\bar{I}^* = VIe^{j\phi} = VI(\cos \phi + j \sin \phi) = P + jQ \text{ [VA]},$$

$$|S| = VI, \quad P = |S| \cos \phi, \quad Q = |S| \sin \phi.$$

$Q > 0$ (current lags) means the circuit absorbs reactive power, inductive; $Q < 0$ (current leads) means it supplies reactive power, capacitive.

For a series branch $Z = R + jX$:

KEY RELATION

$$P = I^2 R, \quad Q = I^2 X, \quad \frac{Q}{P} = \frac{X}{R} = \tan \phi, \quad S = Z I^2.$$

A parallel capacitor across the load voltage V supplies $Q_c = V^2/X_C$, subtracting from what the supply must provide:

KEY RELATION

$$Q_{new} = Q_{old} - Q_c, \quad P_{new} = P, \quad X_C = \frac{V^2}{Q_c}, \quad C = \frac{1}{\omega X_C} = \frac{Q_c}{\omega V^2}.$$

The base P never changes; $|S|$ and the supply current shrink. Past $Q_c = Q_{old}$ the angle crosses to leading – over-correction is not free.

WORKED EXAMPLE · MODULE 2, EXAMPLE 2 (P.27) · CORE · NUMBERS FROM P.28 OF THE SOLVED DECK

For the circuit shown, calculate Q_{old} and C for $PF_{new} = 1$. The load is $Z = 11 + j11 = 15.56\angle 45^\circ$, the supply is $V = 220 \text{ V}$, and the capacitor C is in parallel with Z .

Step 1, the load current:

$$\bar{I}_Z = \frac{\bar{V}}{Z} = \frac{220\angle 0^\circ}{15.56\angle 45^\circ} = 14.14\angle -45^\circ \text{ A.}$$

Step 2, the reactive power to cancel:

$$Q_{old} = VI_Z \sin \phi_{old} = 220 \times 14.14 \times \sin 45^\circ = 2200 \text{ VAR} = I^2 X = 14.14^2 \times 11.$$

Step 3, the capacitor. Unity power factor means $Q_c = Q_{old} = 2200 \text{ VAR}$, so

$$X_C = \frac{V^2}{Q_c} = \frac{220^2}{2200} = 22 \Omega \Rightarrow C = \frac{1}{\omega X_C} = \frac{1}{2\pi(60)(22)} = 120.6 \mu\text{F}.$$

Check. $\bar{I}_C = 220/(22\angle -90^\circ) = 10\angle +90^\circ \text{ A}$, and

$$\bar{I} = \bar{I}_Z + \bar{I}_C = (10 - j10) + (0 + j10) = 10\angle 0^\circ \text{ A},$$

in phase with the voltage, so the power factor is 1.

Balanced three-phase circuits

Module 2 part C.

KEY RELATION

$$\bar{V}_{an} = V\angle 0^\circ, \quad \bar{V}_{bn} = V\angle -120^\circ, \quad \bar{V}_{cn} = V\angle -240^\circ.$$

In a **star** (Y), line voltages are differences of phase voltages. In a **delta** the phase voltages *are* the line voltages, and the phase currents' differences give the line currents:

KEY RELATION

$$\bar{V}_{ab} = \sqrt{3} \bar{V}_a \angle 30^\circ, \quad \bar{I}_a = \bar{I}_{ab} - \bar{I}_{ca} = \sqrt{3} \bar{I}_{ab} \angle -30^\circ.$$

STAR AND DELTA, SIDE BY SIDE

	STAR (Y)	DELTA
Voltages	$V_L = \sqrt{3} V_{ph}$, line leads phase by 30°	$V_L = V_{ph}$
Currents	$I_L = I_{ph}$	$I_L = \sqrt{3} I_{ph}$, line lags phase by 30°
Has a neutral	yes	no
Sums that vanish: $\bar{V}_a + \bar{V}_b + \bar{V}_c = 0$, $\bar{V}_{ab} + \bar{V}_{bc} + \bar{V}_{ca} = 0$, $\bar{I}_a + \bar{I}_b + \bar{I}_c = 0$		

Y- Δ conversion. In general (Tutorial 2), with Z_a, Z_b, Z_c the wye arms at nodes a, b, c and Z_{ab}, Z_{bc}, Z_{ca} the delta sides:

KEY RELATION

$$Z_{ab} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_c}, \quad Z_a = \frac{Z_{ab} Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}}.$$

A delta side is the sum of the pairwise products over the *opposite* wye arm; a wye arm is the product of the two *adjacent* delta sides over their sum. Balanced, both collapse to $Z_\Delta = 3 Z_Y$. Source and load must both be wye to solve a per-phase equivalent; a delta at either end is converted first.

WORKED EXAMPLE · MODULE 2, EXAMPLE 4 (P.60) · CORE · WORKED SOLUTION AGREES WITH THE SOLVED DECK, P.61

In a balanced three-phase circuit, $\bar{V}_{ab} = 173.2\angle 0^\circ$ V. Determine all the phase and line voltages and currents in a Y-connected load having $Z_L = 10\angle 20^\circ \Omega$.

Divide by $\sqrt{3}$ and rotate back by 30° :

$$\bar{V}_{an} = \frac{173.2}{\sqrt{3}} \angle (0^\circ - 30^\circ) = 100 \angle -30^\circ \text{ V},$$

$$\bar{V}_{bn} = 100 \angle -150^\circ, \quad \bar{V}_{cn} = 100 \angle 90^\circ \text{ V},$$

$$\bar{V}_{ab} = 173.2 \angle 0^\circ, \quad \bar{V}_{bc} = 173.2 \angle -120^\circ, \quad \bar{V}_{ca} = 173.2 \angle 120^\circ \text{ V}.$$

In a wye the line current is the phase current:

$$\bar{I}_a = \frac{100 \angle -30^\circ}{10 \angle 20^\circ} = 10 \angle -50^\circ \text{ A}, \quad \bar{I}_b = 10 \angle -170^\circ, \quad \bar{I}_c = 10 \angle 70^\circ \text{ A}.$$

The load is balanced, so $\bar{I}_n = 0$.

Three-phase power, and measuring it

KEY RELATION

$$P_{total} = 3 V_{ph} I_{ph} \cos \theta = \sqrt{3} V_L I_L \cos \theta = \frac{V_L^2}{Z_Y} \cos \theta,$$

$$Q_{total} = 3 V_{ph} I_{ph} \sin \theta = \sqrt{3} V_L I_L \sin \theta = \frac{V_L^2}{Z_Y} \sin \theta.$$

Star or delta alike; θ is the **phase** impedance angle, and Z_Y is the per-phase wye impedance (its magnitude). For a delta load, work per phase with the phase quantities, $V_{ph} = V_L$ and $I_{ph} = I_L / \sqrt{3}$:

$$P_\Delta = 3 I_{ph}^2 R_\Delta = \frac{3 V_L^2}{|Z_\Delta|} \cos \theta, \quad Q_\Delta = 3 I_{ph}^2 X_\Delta, \quad I_{ph} = \frac{V_L}{|Z_\Delta|}.$$

Constant power. The three phases' instantaneous powers each pulse at 2ω , but the pulses are 120° apart and cancel, so the total

$$p(t) = p_a(t) + p_b(t) + p_c(t) = 3 V_{ph} I_{ph} \cos \theta$$

is constant (RMS values) – why a three-phase machine runs with a steady torque where a single-phase one shakes at twice line frequency.

The trap: $S_{total} = 3 \bar{V}_{ph} \bar{I}_{ph}^* \neq \sqrt{3} \bar{V}_L \bar{I}_L^*$ – the magnitudes agree but the angles do not, so build S_{total} from phase quantities.

A wattmeter reads $V I \cos \phi$: voltage coil across the terminals, current coil in series in the branch. Three-wattmeter (wye load, needs a neutral): total $P_A + P_B + P_C$. Two-wattmeter (delta load, no neutral): total $P_1 + P_2$.

WORKED EXAMPLE · MODULE 2, EXAMPLE 7 (P.66) · CORE · CROSS-CHECKED TWO WAYS IN THE DECK

A wye source with a 440 V line voltage supplies load 1, 4 Ω resistors in wye, and load 2, 6 Ω resistors in delta. Find each load's phase voltage and line current, the source line current, and the power delivered.

Voltages. $V_{ph,1} = 440/\sqrt{3} = 254.0$ V; $V_{ph,2} = V_L = 440$ V (delta).

Currents. $I_{ph,1} = 254.0/4 = 63.51$ A = $I_{L,1}$; $I_{ph,2} = 440/6 = 73.33$ A, $I_{L,2} = \sqrt{3} \times 73.33 = 127.0$ A.

Source line current. Both loads resistive, so their line currents are in phase with $\bar{V}_{an}$ and add directly: $I_L = 63.51 + 127.0 = 190.5$ A. Check via the per-phase equivalent: $Z_Y = Z_\Delta/3 = 2 \Omega$ in parallel with the 4 Ω wye load gives $4||2 = 1.333 \Omega$, so $254.0/1.333 = 190.5$ A.

Power. Resistive loads, $\cos \theta = 1$: $P = \sqrt{3} V_L I_L \cos \theta = \sqrt{3} \times 440 \times 190.5 = 145.2$ kW. Check per load: load 1 dissipates $3 \times 254.0^2/4 = 48.4$ kW, load 2 dissipates $3 \times 440^2/6 = 96.8$ kW, summing to 145.2 kW.

Single-phase R, L and C

Every branch sees the same source voltage E , so currents add as phasors:

KEY RELATION

$$\bar{I}_R = \frac{E}{R} \angle 0^\circ, \quad \bar{I}_L = \frac{E}{X_L} \angle -90^\circ, \quad \bar{I}_C = \frac{E}{X_C} \angle +90^\circ, \quad X_L = 2\pi f L, \quad X_C = \frac{1}{2\pi f C},$$

$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C = I_R - j(I_L - I_C), \quad P = E I_R, \quad Q = E(I_L - I_C), \quad PF = \frac{I_R}{|\bar{I}|}.$$

WORKED EXAMPLE · 1-PH LAB MANUAL, TABLE 1.6 STATION 1 · CORE ·

Station 1: $R = 171.43 \Omega$, $L = 0.8$ H, $C = 2.2 \mu\text{F}$, $E = 120$ V at 60 Hz, ideal elements. $X_L = 2\pi(60)(0.8) = 301.6 \Omega$ and $X_C = 1/(2\pi(60)(2.2 \times 10^{-6})) = 1205.7 \Omega$, so $I_R = 0.700$ A, $I_L = 0.398$ A and $I_C = 0.0995$ A.

STATION 1, THE BASE CASES AND RLC (IDEAL)

ON	A/M5	P (METER)	Q	S	PF
R	0.700 A	84.0 W (P1)	0	84.0 VA	1.000
L	0.398 A	0 (B/M11)	+47.7 VAR	47.7 VA	0 (lags 90°)
C	0.100 A	0 (B/M11)	-11.9 VAR	11.9 VA	0 (leads 90°)
RLC	0.761 A	84.0 W (P1)	+35.8 VAR	91.3 VA	0.920 lag

Magnetic circuits

Module 3: every machine in Modules 4–7 is the same object – a magnetic circuit with windings on it.

Fields, MMF and reluctance

A coil of N turns carrying I drives flux around a core exactly as an EMF drives current through a resistive network:

KEY RELATION

$$B = \frac{\phi}{A} [\text{T}], \quad H = \frac{NI}{\ell} [\text{A}\cdot\text{t/m}], \quad \mathcal{F} = NI = H\ell = \mathcal{R}\phi,$$

$$\mathcal{R} = \frac{\ell}{\mu A} \left[\frac{\text{A}\cdot\text{t}}{\text{Wb}} \right], \quad \mu = \mu_r \mu_0, \quad \mu_0 = 4\pi \times 10^{-7} \text{ H/m}.$$

ℓ is the *mean* path, along the centre line of the core, and A the cross-section. One loop carries the same flux in every segment, so B is largest where the core is thinnest. Reluctance is constant in air ($\mu_r = 1$) and not constant in steel.

THE ANALOGY

MAGNETIC	ELECTRIC
MMF $\mathcal{F} = NI$	EMF e
flux ϕ	current i
reluctance $\ell/(\mu A)$	resistance $\ell/(\sigma A)$
$B = \phi/A, H = \mathcal{F}/\ell$	$J = i/A, E = e/\ell$

Series reluctances add and parallel ones add as reciprocals. Round any loop the MMF drops equal the sources (magnetic KVL, $\sum NI = \sum \mathcal{R}\phi$); the fluxes into a junction sum to zero (magnetic KCL). Air's permeability is about a thousand times smaller than iron's, so a millimetre

of gap can out-reluctance the whole iron path round it: in a machine almost all the MMF is spent crossing the gap.

B-H, hysteresis and core loss

$B = \mu H$ bends because the domains align until the core saturates: a linear region, a knee, then saturation, where $\mu = B/H$ collapses toward air's value, $\mathcal{R}$ grows, and doubling the MMF buys far less than double the flux. Machines run **near the knee**. Under AC the core traces a hysteresis loop: residual B_r at $H = 0$, coercive H_c to bring B back to zero. The loop's **area** is energy lost per cycle per unit volume. Soft materials (narrow loop) make cores; hard ones (wide loop, large B_r) make permanent magnets. A smaller amplitude traces a nested minor loop.

A changing flux also induces EMF inside the iron, which drives circulating eddy currents. Thin laminations (0.35–0.5 mm, varnish-insulated) along the flux direction break those loops.

KEY RELATION

$$P_h = k_h V B_{max}^n f \quad (n = 1.5-2.5), \quad P_e = k_e V (f t B_{max})^2, \quad P_{core} = P_h + P_e.$$

V is the core volume and t the lamination thickness. Doubling f doubles P_h and quadruples P_e ; thinner laminations cut only P_e .

The analogy breaks in two more places. **Leakage**: some flux leaves the core and returns through air without linking the far winding, so more MMF is needed – and why a real transformer has a leakage reactance. **Fringing**: flux at a gap spreads sideways, so the gap's effective area exceeds the pole face,

$$A_g = (w + \ell_g)(d + \ell_g) \quad \text{or a stated percentage,}$$

and $B_g < B_{core}$. Apply it to the gap only.

Solving a core

Split the core into segments, take $\mathcal{R} = \ell/(\mu A)$ for each, combine them as a network, then $\phi = \mathcal{F}/\mathcal{R}$ and $B = \phi/A$. A gap shortens its leg's iron by ℓ_g . A core with two windings: their MMFs add or subtract by the winding senses (right-hand rule). With no μ_r given, work from the B - H curve instead: $H = NI/\ell$, read B , then $\phi = BA$; going the other way, $\phi \rightarrow B \rightarrow H$ from the curve, then $NI = \sum H\ell$.

WORKED EXAMPLE · MODULE 3, EXAMPLE 1 · CORE · PP.28--32

$N = 250$, $\mu_r = 3000$, depth 10 cm, no gap, 2 A. Mean paths 37.5, 35, 37.5 and 35 cm:

$$\mathcal{R} = 9947.184 \times 2 + 9284.038 + 6189.359 = 35\,367.765 \text{ A}\cdot\text{t/Wb},$$

$$\phi = \frac{500}{35\,367.765} = 0.014137 \text{ Wb}, \quad B = 1.414 \text{ T}.$$

Now cut a 0.5 mm gap, area +2.5% for fringing, and hold the same flux: $\mathcal{R}_g = 38\,818.279$, the gapped leg's iron $\mathcal{R}'_2 = 9270.775$, so $\mathcal{R} = 74\,172.781$ and $I = \mathcal{R}\phi/N = 4.195 \text{ A}$ – about 2.1× the current, for half a millimetre of air.

WORKED EXAMPLE · MODULE 3, EXAMPLE 2 · DC MACHINE, TWO GAPS · P.33

Stator 185 cm, 428.36 cm²; rotor 10 cm, 424.12 cm²; two 0.5 mm gaps of 450 cm²; $\mu_r = 2500$; $NI = 600$.

$$\mathcal{R}_s = 13\,747, \quad \mathcal{R}_r = 750.5, \quad \mathcal{R}_g = 8842 \text{ each},$$

$$\mathcal{R} = 13\,747 + 750.5 + 2(8842) = 32\,182 \text{ A}\cdot\text{t/Wb},$$

$$\phi = 600/32\,182 = 18.64 \text{ mWb}, \quad B_g = 0.414 \text{ T}, \quad B_s = 0.435 \text{ T}.$$

The flux crosses the gap *twice*; with one gap you get 25.7 mWb, the usual slip. The published solution (slide 36) adds $\mathcal{R}_r$ twice instead of $\mathcal{R}_g$ twice and gets 24 090 and 0.0249 Wb.

Induced EMF and force

KEY RELATION

$$e = N \frac{d\phi}{dt} = \frac{d\lambda}{dt}, \quad \lambda = N\phi; \quad e = (\vec{v} \times \vec{B}) \cdot \vec{\ell} = v\ell B; \quad \vec{F} = i(\vec{\ell} \times \vec{B}), \quad F = i\ell B.$$

Lenz: the induced current's flux opposes the *change* in flux, which is the minus sign in $e = -N d\phi/dt$ – a statement about direction, not size. A conductor moving through a field generates ($e = v\ell B$); a conductor carrying current in a field is pushed ($F = i\ell B$). Every machine does both at once. Example: 100 turns, $\phi = 0.01 \sin 377t$ gives $e = 377 \cos 377t \text{ V}$.

CHECK YOURSELF

A core is already past the knee of its B - H curve. You double the coil current. What happens to the flux and to the reluctance?

Answer. The flux rises by much less than a factor of two: past the knee μ has collapsed toward air's value, so $\mathcal{R} = \ell/(\mu A)$ has grown, and the extra MMF works against a larger opposition than the first half did.

Transformers

Module 4. The worked solutions on the slides are blank, so every answer in this section is ours.

The ideal transformer

Two coils linked by the same flux on an iron core. Ideal means $\mu \rightarrow \infty$ (so $\mathcal{R} = 0$), no leakage, and no core or winding losses. With $\phi = \phi_m \sin \omega t$, Faraday gives each winding's RMS voltage:

KEY RELATION

$$V = \frac{\omega \phi_m N}{\sqrt{2}} = 4.44 f \phi_m N, \quad a = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1},$$

$$S_1 = \bar{V}_1 \bar{I}_1^* = \bar{V}_2 \bar{I}_2^* = S_2, \quad Z'_2 = a^2 Z_2.$$

The current ratio comes from Ampère's law with $\mathcal{R} = 0$: $N_1 i_1 = N_2 i_2$. The impedance seen through the transformer scales by the square because the voltage scales by a and the current by $1/a$; referring with a single factor of a is the commonest error in this module. **Dots:** if the dotted primary terminal is the higher potential, so is the dotted secondary one; current *into* the primary dot leaves *out of* the secondary dot. Uses: stepping up ($a < 1$) or down ($a > 1$), isolation ($a = 1$), measurement, and impedance matching for maximum power transfer. Real units run 92% (small) to 99% (large) efficient.

WORKED EXAMPLE · MODULE 4, EXAMPLE (P.13) · CORE · OURS

A 240/120 V, 60 Hz ideal transformer rated 5 kVA.

Answer. $a = 240/120 = 2$. Rated currents $5000/240 = 20.83$ A (primary) and $5000/120 = 41.67$ A (secondary). Delivering 3.2 kW at 0.8 lagging and rated secondary voltage: $S = 3.2/0.8 = 4$ kVA, so $I_2 = 4000/120 = 33.33$ A and $I_1 = I_2/a = 16.67$ A.

The real transformer and its equivalent circuit

Each winding gets a series resistance (R_1, R_2 : I^2R copper loss) and a leakage reactance (X_1, X_2 : flux that links one winding only). Finite permeability and core loss become a shunt branch $R_c \parallel jX_m$ across the induced voltage E_1 : the exciting current $\bar{I}_\phi = \bar{I}_c + \bar{I}_m$ splits into a core-loss part through R_c and a magnetizing part through X_m . An ideal transformer sits between E_1 and E_2 .

Referral. Move the secondary across the ideal transformer to the primary side and the ideal transformer disappears:

KEY RELATION

$$R'_2 = a^2 R_2, \quad X'_2 = a^2 X_2, \quad Z'_L = a^2 Z_L, \quad \bar{V}'_2 = a \bar{V}_2, \quad \bar{I}'_2 = \bar{I}_2 / a.$$

To the secondary side, divide impedances by a^2 instead. Then solve it as an ordinary circuit, in phasors.

Approximate circuits. The exciting current is 3–5% of rated, and the series drops are small, so $V_1 \approx E_1$. Moving the shunt branch to the terminals lets the series elements combine:

KEY RELATION

$$R_{e1} = R_1 + a^2 R_2, \quad X_{e1} = X_1 + a^2 X_2, \quad \bar{V}_1 = a \bar{V}_2 + \bar{I}'_2 (R_{e1} + jX_{e1}).$$

That is the 1st approximation; the 2nd drops the shunt branch altogether, which is valid unless efficiency or the exciting current itself is the question.

WORKED EXAMPLE · MODULE 4, EXAMPLE (P.24) · CORE · OURS

25 kVA, 440/220 V, 60 Hz; $R_1 = 0.16$, $R_2 = 0.04$, $X_1 = 0.32$, $X_2 = 0.08$, $R_c = 270$, $X_m = 100 \Omega$ (shunt on the HV side as drawn). It delivers 20 kW at 0.8 lagging at rated voltage to a load on the LV side. Find the primary terminal voltage.

$a = 2$, so $R'_2 = 0.16$, $X'_2 = 0.32 \Omega$. Load current $\bar{I}_2 = 20000 / (0.8 \times 220) = 113.64 \angle -36.87^\circ$ A, referred $\bar{I}'_2 = 56.82 \angle -36.87^\circ$ A, with $\bar{V}'_2 = 440 \angle 0^\circ$ V.

$$\begin{aligned} \bar{E}_1 &= \bar{V}'_2 + \bar{I}'_2(0.16 + j0.32) = 458.27 \angle 1.14^\circ \text{ V}, \\ \bar{I}_\phi &= \frac{\bar{E}_1}{270} + \frac{\bar{E}_1}{j100} = 4.887 \angle -68.54^\circ \text{ A}, \quad \bar{I}_1 = \bar{I}'_2 + \bar{I}_\phi = 61.03 \angle -39.28^\circ \text{ A}, \\ \bar{V}_1 &= \bar{E}_1 + \bar{I}_1(0.16 + j0.32) = 478.4 \angle 2.16^\circ \text{ V}. \end{aligned}$$

The approximate circuit, $\bar{V}_1 = 440 + \bar{I}'_2(0.32 + j0.64)$, gives 476.7 V.

Finding the parameters: two tests

Open-circuit test – rated voltage on the LV side, HV open; measure P_{oc} , V_{oc} , I_{oc} on the LV side. The series branch carries only the tiny exciting current, so all of P_{oc} is core loss and the measurement sees only the shunt branch. The LV side is chosen because a rated-voltage source is easier there, the meters are cheaper, and the small current reads more accurately.

KEY RELATION

$$R_c = \frac{V_{oc}^2}{P_{oc}}, \quad |Y_\phi| = \frac{I_{oc}}{V_{oc}}, \quad G_c = \frac{1}{R_c}, \quad B_m = \sqrt{|Y_\phi|^2 - G_c^2}, \quad X_m = \frac{1}{B_m}.$$

Short-circuit test – LV shorted, HV on a low variable voltage raised until rated current flows; measure P_{sc} , V_{sc} , I_{sc} on the HV side. The voltage is so low that the core flux and core loss are negligible, so all of P_{sc} is the rated copper loss, and the shunt branch drops out.

KEY RELATION

$$R_{e1} = \frac{P_{sc}}{I_{sc}^2}, \quad |Z_{e1}| = \frac{V_{sc}}{I_{sc}}, \quad X_{e1} = \sqrt{|Z_{e1}|^2 - R_{e1}^2}, \\ R_1 = a^2 R_2 = \frac{1}{2} R_{e1}, \quad X_1 = a^2 X_2 = \frac{1}{2} X_{e1}$$

when the design splits the losses and leakage equally. Slide 35 puts the short-circuit voltage at “20–30% of rated”, but its own example (p.40) uses 48 V on a 2400 V winding, which is 2%.

Regulation and efficiency

KEY RELATION

$$\varepsilon = \frac{|V_{2,nl}| - |V_{2,fl}|}{|V_{2,fl}|} \times 100\% = \frac{|V_1|/a - |V_2|}{|V_2|} \times 100\%,$$

with the magnetizing branch usually neglected. An ideal transformer has $\varepsilon = 0$. Copper loss grows with the square of load current, while core loss is set by the flux and stays put from no load to full load. At a fraction x of full load:

KEY RELATION

$$\eta = \frac{P_{out}}{P_{out} + P_{core} + P_{cu}} = \frac{x S \cos \theta}{x S \cos \theta + P_{core} + x^2 P_{cu,fl}}, \\ \eta_{max} \text{ where } x^2 P_{cu,fl} = P_{core}, \text{ i.e. } x = \sqrt{P_{core}/P_{cu,fl}}.$$

WORKED EXAMPLE · MODULE 4, EXAMPLE (P.40) · CORE · OURS

50 kVA, 2400/240 V, 60 Hz. Open circuit on the LV side: 240 V, 5.4 A, 186 W. Short circuit on the HV side: 48 V, 20.8 A, 620 W. (a) The approximate circuit referred to the primary. (b) Regulation and efficiency at rated load, 0.8 lagging, rated secondary voltage.

Shunt branch (LV side). $R_{c2} = 240^2/186 = 309.7 \Omega$; $|Y| = 5.4/240 = 0.0225 \text{ S}$, $G = 0.003229 \text{ S}$, $B = 0.02227 \text{ S}$, so $X_{m2} = 44.91 \Omega$. Referred to the HV side, multiply by $a^2 = 100$.

Series branch (HV side). $R_{e1} = 620/20.8^2 = 1.433 \Omega$, $|Z_{e1}| = 48/20.8 = 2.308 \Omega$, $X_{e1} = 1.809 \Omega$.

Regulation. $I_1 = 50000/2400 = 20.83 \angle -36.87^\circ \text{ A}$, so $\bar{V}_1 = 2400 + \bar{I}_1(1.433 + j1.809) = 2446.5 \angle 0.29^\circ \text{ V}$ and $\varepsilon = 46.5/2400 = 1.94\%$.

Efficiency. $P_{out} = 40 \text{ kW}$, $P_{cu} = 20.83^2 \times 1.433 = 622 \text{ W}$, $P_{core} = 186 \text{ W}$: $\eta = 40000/40808 = 98.02\%$. Peak efficiency at $x = \sqrt{186/622} = 0.547$, i.e. 54.7% load.

CHECK YOURSELF

A transformer has $a = 10$. A 5Ω load sits on the secondary. What impedance does the primary source see?

Answer. $Z_1 = a^2 Z_2 = 100 \times 5 = 500 \Omega$, not 50Ω .

DC machines

Module 5.

Field winding on the stator, armature on the rotor – the opposite of every AC machine in the next two sections. A commutator mechanically rectifies: the armature carries AC, the terminals see DC. Two equations, sharing one constant K set by the winding geometry, carry the module:

KEY RELATION

$$E_a = K\phi\omega, \quad T = K\phi I_a, \quad E_a I_a = T\omega.$$

$$\text{motor: } V_t = E_a + I_a R_a \quad \text{generator: } V_t = E_a - I_a R_a.$$

At standstill $E_a = 0$, so $I_a = V_t/R_a$ is many times rated – hence starting resistance, needed only until E_a grows enough to limit the current itself. Since

KEY RELATION

$$\omega = \frac{V_t - I_a R_a}{K\phi},$$

weakening the field *raises* speed, because ϕ sits in the denominator. Field-to-armature connection sets the machine type: separately excited (ϕ fixed by an independent supply), shunt (field across the armature terminals, speed droops only slightly with load), series (field in series with the armature, so flux rises with load current and torque goes as I_a^2 – large low-speed torque, but an unloaded series motor has almost no flux and can run away), and compound (both fields, between the two).

CHECK YOURSELF

A shunt DC motor runs at steady speed on a fixed terminal voltage. You reduce the field current. What happens to the speed?

Answer. The speed rises: with V_t fixed, $\omega = (V_t - I_a R_a)/(K\phi)$, and ϕ is in the denominator.

CHECK YOURSELF

A DC motor with $R_a = 0.5 \Omega$ is connected directly to a 250 V supply at standstill. Estimate the armature current at the instant of connection.

Answer. At standstill $E_a = 0$, so $I_a = V_t/R_a = 250/0.5 = 500$ A, many times rated. As the machine accelerates E_a grows in proportion to speed and the current falls to $(V_t - E_a)/R_a$.

Synchronous machines

Module 6.

DC-excited field winding on the rotor, three-phase armature on the stator – the reverse of a DC machine, no commutator needed. The rotating stator field turns only at synchronous speed;

the rotor must match it exactly or average torque is zero, so the machine produces no starting torque on its own:

KEY RELATION

$$n_s = \frac{120f}{p} [\text{rpm}], \quad \bar{E}_f = \bar{V}_t + jX_s \bar{I}_a \text{ (generator)}, \quad P = \frac{3E_f V_t}{X_s} \sin \delta.$$

δ is not free: it settles wherever the electrical power out must match the mechanical power in. Power rises with δ only to $\delta = 90^\circ$ (the maximum deliverable power is $3E_f V_t / X_s$, the steady-state stability limit); past it the electrical power the machine can deliver falls while the mechanical input keeps rising, so the rotor accelerates away and loses synchronism.

On a stiff grid (V_t, f fixed, real power set by the prime mover), the field current instead controls reactive power: over-excite and the machine supplies Q with a lagging armature current; under-excite and it absorbs Q with a leading current. **Watch the current direction:** earlier sections read Q from a load, where a lagging current means the load absorbs Q ; here $\bar{I}_a$ is defined out of the machine, so the same lagging current means the machine *delivers* Q – same physics, opposite bookkeeping. A synchronous motor deliberately over-excited to supply Q to a plant is a synchronous condenser: power-factor correction done with a machine instead of a capacitor.

WORKED EXAMPLE · PER-UNIT EXCITATION

Take $V_t = 1\angle 0^\circ$ pu, $X_s = 1$ pu, $P = 0.5$ pu and an over-excited $E_f = 1.5$ pu. Then $\delta = 19.5^\circ$ and

$$\bar{I}_a = \frac{\bar{E}_f - \bar{V}_t}{jX_s} = 0.649\angle -39.6^\circ \text{ pu},$$

a lagging current, and $\bar{S} = \bar{V}_t \bar{I}_a^* = 0.5 + j0.41$ pu: reactive power flows out of the machine. Drop E_f to 1.05 pu and $\bar{I}_a$ swings to $+8.7^\circ$, leading, with $Q = -0.08$ pu – absorbed.

Induction motors

Module 7.

If the rotor turned at synchronous speed it would see a stationary field, no flux change, no induced current, no torque – it must run slower, by the slip:

KEY RELATION

$$s = \frac{n_s - n}{n_s}, \quad \frac{R_2}{s} = R_2 + R_2 \frac{1-s}{s}.$$

The first piece is genuine rotor resistance, dissipating rotor copper loss; the second is not a resistance at all, and the power it absorbs is where mechanical power leaves the circuit. At

$s = 1$ it vanishes (a stalled motor delivers no mechanical power); as $s \rightarrow 0$ it grows without bound (almost all air-gap power is converted, not dissipated). Torque as a function of slip:

KEY RELATION

$$T = \frac{3}{\omega_s} \cdot \frac{V_1^2(R_2/s)}{(R_1 + R_2/s)^2 + (X_1 + X_2)^2}, \quad s_{max} = \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}}.$$

R_2 cancels when s_{max} is substituted back: rotor resistance moves *where* breakdown torque sits, not *how high* – the basis of wound-rotor starting. Torque goes as V_1^2 at every slip, so a lower supply voltage scales the whole curve down without moving the breakdown slip – reduced-voltage starting is gentle on the supply and weak on torque. A third handle, supply frequency, moves n_s itself: what a variable-frequency drive does. Parameter tests mirror the transformer's: no-load (rotor branch effectively open) gives the magnetizing branch and rotational losses; blocked-rotor (reduced voltage, $s = 1$) gives the series resistances and leakage reactances.

CHECK YOURSELF

A 4-pole 60 Hz induction motor runs at 1728 rpm. Compute the slip and the fraction of air-gap power lost as rotor copper loss.

Answer. $n_s = 120 \times 60/4 = 1800$ rpm, $s = (1800 - 1728)/1800 = 0.04$: 4% of the air-gap power is rotor copper loss, the other 96% becomes mechanical power.

Practice problems

From the filed decks and the lessons' check-yourself questions, same numbers.

1. (**Module 2, Example 1, p.22**) A load on a 79.7 kV busbar is $Z = 80 + j60 \Omega$. Find the real and reactive power it absorbs and its power factor.

Answer: $\bar{I} = 797 \angle -36.87^\circ$ A, $S = 63.52 \angle 36.87^\circ$ MVA, so $P = 50.82$ MW, $Q = 38.11$ MVAR, $PF = 0.8$ **lagging**.

2. A balanced delta load of $30 \angle 40^\circ \Omega$ per phase sits on a 208 V line. What impedance goes into the per-phase equivalent circuit, and what voltage does that circuit see?

Answer: $Z_Y = Z_\Delta/3 = 10 \angle 40^\circ \Omega$, the angle unchanged. Drawn line-to-neutral, it sees $208/\sqrt{3} = 120.1$ V.

3. (**PS2 P2**) A balanced delta load of $12 + j9 \Omega$ per phase on a 208 V, 60 Hz source. Find the line current, P , Q and the power factor.

Answer (official): $I_{ph} = 208/15 = 13.87 \text{ A}$, $\bar{I}_a = 24.02 \angle -66.87^\circ \text{ A}$ ($\bar{V}_{ab}$ reference), $P = 6923 \text{ W}$, $Q = 5192 \text{ VAR}$, $PF = 0.8$ lagging.

4. (PS2 P5) An 18Ω delta in parallel with a 12Ω wye, both resistive, fed from 346.41 V line-to-line through $j3 \Omega$ per phase. Find the source current and the power it supplies.

Answer (official): $18 \Omega \Delta \rightarrow 6 \Omega \text{ Y}$, $6 \parallel 12 = 4 \Omega$; per phase $200/(4+j3)$, so $\bar{I}_s = 40 \angle -36.87^\circ \text{ A}$, $P = 19.2 \text{ kW}$ (all in the loads) and $Q = 14.4 \text{ kVAR}$ (all in the line).

5. (PS2 P7) 15 kW at 0.6 lagging in parallel with 10 kVA at 0.8 leading, on 480 V . Find the combined power factor, the line current, and the delta capacitor bank that makes it unity.

Answer (official): $S = (15 + j20) + (8 - j6) = 26.93 \angle 31.33^\circ \text{ kVA}$, $PF = 0.854$ lagging, $I_L = 32.4 \text{ A}$. Cancelling 14 kVAR across three delta legs at 480 V needs $X_C = 480^2/4667 = 49.37 \Omega$ per leg, and the line current falls to 27.66 A .

6. (PS3 P1, unofficial) $\mu_r = 800$, 500 turns, $\phi = 5 \text{ mWb}$. *Answer (ours, from the lesson):* $\mathcal{R} = 59\,683 + 72\,946 + 119\,366 = 251\,995 \text{ A}\cdot\text{t/Wb}$; $I = 2.52 \text{ A}$; $B_{top} = 0.667 \text{ T}$, $B_{right} = 2.0 \text{ T}$.

7. (PS3 P2, unofficial) Three legs, 300 turns at 1 A on the centre, $\mu_r = 1500$, gaps $+5\%$. The text gives the left gap 0.050 cm and the right 0.070 cm ; the figure has them the other way round.

Answer (ours, text values): $\phi_c = 1.468 \text{ mWb}$, $\phi_L = 0.787$, $\phi_R = 0.681 \text{ mWb}$; $B_{gL} = 0.214 \text{ T}$, $B_{gR} = 0.185 \text{ T}$. The smaller gap carries more flux.

8. (PS3 P3, unofficial) *Answer (ours):* $\mathcal{R} = 76\,630$; aiding $\phi = 6.52 \text{ mWb}$, opposing 1.30 mWb . Say which you read from the winding senses.

9. (PS3 P5, unofficial) 2 A upward along 1 m of wire in 0.5 T pointing right. *Answer (ours):* $F = i\ell B = 1 \text{ N}$, into the page.

10. Mean path 60 cm , 16 cm^2 , $\mu_r = 2000$, a 1 mm gap at $+4\%$, 400 turns, 1.0 T in the core. Find the current, the gap flux density, and the current without the gap.

Answer: $\mathcal{R}_m = 148\,959$, $\mathcal{R}_g = 478\,230$, $I = 2.51 \text{ A}$, $B_g = 0.962 \text{ T}$; without the gap 0.60 A .

11. A transformer has $a = 10$. A 5Ω load sits on the secondary. Predict the impedance the primary source sees.

Answer: $Z_1 = a^2 Z_2 = 100 \times 5 = 500 \Omega$, not 50Ω .

12. A shunt DC motor runs at steady speed on a fixed terminal voltage. You reduce the field current. Predict what happens to the speed, and name the term that drives it.

Answer: The speed rises: with V_t fixed, $\omega = (V_t - I_a R_a)/(K\phi)$, and ϕ is in the denominator.

13. A synchronous generator on a stiff grid delivers 5 MW . You raise its field current and leave the prime mover alone. Predict what happens to the real and reactive power.

Answer: Real power stays at 5 MW – the prime mover sets it and the grid holds V_t and f . Raising E_f makes the machine supply more reactive power, and δ falls to keep P constant.

14. A 4-pole 60 Hz induction motor runs at 1728 rpm . Compute the slip and the fraction of air-gap power lost as rotor copper loss.

Answer: $n_s = 120 \times 60/4 = 1800$ rpm, $s = (1800 - 1728)/1800 = 0.04$, so 4% of the air-gap power is rotor copper loss.

15. Two identical wound-rotor induction motors differ only in added rotor resistance. Compare their starting torque and their breakdown torque.

Answer: The one with added resistance has the higher starting torque, because $s_{max} = R_2/\sqrt{R_1^2 + X^2}$ moves toward $s = 1$. Their breakdown torques are identical: R_2 cancels out of the torque expression once s_{max} is substituted back in.

Lesson sources: lessons/ECE260/claude_lessons/ece260-ac-circuits/src/ece260_ac_circuits.jsx and lessons/ECE260/claude_lessons/ece260-course-overview/src/ece260_course_overview.jsx, read 2026-09-27; ECE 260 Module 1 and 2 slides and the single-phase lab manual (S. Pirooz Azad, Fall 2026), filed under lessons/ECE260/materials/. Extended 2026-10-05 from lessons/ECE260/claude_lessons/ece260-magnetic-circuits/src/ece260_magnetic_circuits.jsx and its companion, the Module 3 and 4 slides, Tutorial 2, and the Problem Set 2 solutions; Problem Set 3 and the Module 4 examples have no published solutions, so their answers here are ours. The course bans generative AI on assessments: these notes are study material, never an answer to graded work.