

AC circuits and three-phase power

Key-points companion for <https://lessons.ihsan.cc/ece260/ece260-ac-circuits/> – formulas, the highest-value worked examples in full, and practice with answers.

THE CONVENTIONS, USED EXACTLY

Capitals are RMS. $S = \bar{V}\bar{I}^*$, with the conjugate on the *current*; $Q > 0$ absorbs (lagging, inductive), $Q < 0$ supplies. The sequence is $0^\circ, -120^\circ, -240^\circ$ with phase A the reference. The power-factor angle is ϕ in the single-phase parts and θ in the three-phase parts: the same angle.

The power system

Generation, transmission, distribution: 13.8–24 kV, 230–765 kV, 69–138 kV, 2.2–46 kV and 120–480 V. Meshed for reliability, radial for cost.

Phasors, impedance, and instantaneous power

ON THE FORMULA SHEET

$$v(t) = \sqrt{2} V \cos \omega t, \quad i(t) = \sqrt{2} I \cos(\omega t - \phi), \quad V = \frac{V_{max}}{\sqrt{2}}; \quad \bar{V} = V \angle 0^\circ, \quad \bar{I} = I e^{-j\phi} = I \angle -\phi.$$

A lagging current has a negative angle: $\phi > 0$ puts $\bar{I}$ below the real axis.

ON THE FORMULA SHEET

$$Z = \frac{V \angle \theta_v}{I \angle \theta_i} = |Z| \angle \theta, \quad j\omega L = jX_L, \quad -j \frac{1}{\omega C} = -jX_C, \quad Z_{RLC} = R + j(X_L - X_C).$$

Instantaneous power:

ON THE FORMULA SHEET

$$p(t) = P(1 + \cos 2\omega t) + Q \sin 2\omega t, \quad P = VI \cos \phi \text{ [W]}, \quad Q = VI \sin \phi \text{ [VAR]}.$$

P is never negative; Q averages exactly zero – a pure L or C borrows energy and returns it each cycle.

ON THE FORMULA SHEET

$$PF = \cos \phi, \quad RF = \sin \phi, \quad \phi > 0 \text{ for lagging current.}$$

Lagging is inductive, leading capacitive; always quote which.

Complex power and power-factor correction

ON THE FORMULA SHEET

$$S = \bar{V}\bar{I}^* = VIe^{j\phi} = VI(\cos \phi + j \sin \phi) = P + jQ \text{ [VA]},$$

$$|S| = VI, \quad P = |S| \cos \phi, \quad Q = |S| \sin \phi.$$

For a series branch $Z = R + jX$:

ON THE FORMULA SHEET

$$P = I^2 R, \quad Q = I^2 X, \quad \frac{Q}{P} = \frac{X}{R} = \tan \phi, \quad S = Z I^2.$$

A parallel capacitor across the load voltage V supplies $Q_c = V^2/X_C$, subtracting from what the supply must provide:

ON THE FORMULA SHEET

$$Q_{new} = Q_{old} - Q_c, \quad P_{new} = P, \quad X_C = \frac{V^2}{Q_c}, \quad C = \frac{1}{\omega X_C} = \frac{Q_c}{\omega V^2}.$$

The base P never changes; $|S|$ and the supply current shrink. Past $Q_c = Q_{old}$ the angle crosses to leading – over-correction is not free.

WORKED EXAMPLE · MODULE 2, EXAMPLE 2 (P.27) · CORE · NUMBERS FROM P.28 OF THE SOLVED DECK; STEPS AND CHECKS OURS

For the circuit shown, calculate Q_{old} and C for $PF_{new} = 1$. The load is $Z = 11 + j11 = 15.56\angle 45^\circ$, the supply is $V = 220 \text{ V}$, and the capacitor C is in parallel with Z .

Step 1, the load current:

$$\bar{I}_Z = \frac{\bar{V}}{Z} = \frac{220\angle 0^\circ}{15.56\angle 45^\circ} = 14.14\angle -45^\circ \text{ A.}$$

Step 2, the reactive power to cancel:

$$Q_{old} = VI_Z \sin \phi_{old} = 220 \times 14.14 \times \sin 45^\circ = 2200 \text{ VAR} = I^2 X = 14.14^2 \times 11.$$

Step 3, the capacitor. Unity power factor means $Q_c = Q_{old} = 2200 \text{ VAR}$, so

$$X_C = \frac{V^2}{Q_c} = \frac{220^2}{2200} = 22 \Omega \Rightarrow C = \frac{1}{\omega X_C} = \frac{1}{2\pi(60)(22)} = 120.6 \mu\text{F}.$$

Check. $\bar{I}_C = 220/(22\angle -90^\circ) = 10\angle +90^\circ \text{ A}$, and

$$\bar{I} = \bar{I}_Z + \bar{I}_C = (10 - j10) + (0 + j10) = 10\angle 0^\circ \text{ A},$$

in phase with the voltage, so the power factor is 1.

Balanced three-phase circuits

ON THE FORMULA SHEET

$$\bar{V}_{an} = V\angle 0^\circ, \quad \bar{V}_{bn} = V\angle -120^\circ, \quad \bar{V}_{cn} = V\angle -240^\circ.$$

In a **star (Y)**, line voltages are differences of phase voltages. In a **delta** the phase voltages *are* the line voltages, and the phase currents' differences give the line currents:

ON THE FORMULA SHEET

$$\bar{V}_{ab} = \sqrt{3} \bar{V}_a \angle 30^\circ, \quad \bar{I}_a = \bar{I}_{ab} - \bar{I}_{ca} = \sqrt{3} \bar{I}_{ab} \angle -30^\circ.$$

TABLE 3 *the two connections*

	STAR (Y)	DELTA
Voltages	$V_L = \sqrt{3} V_{ph}$, line leads phase by 30°	$V_L = V_{ph}$
Currents	$I_L = I_{ph}$	$I_L = \sqrt{3} I_{ph}$, line lags phase by 30°
Has a neutral	yes	no
Sums that vanish: $\bar{V}_a + \bar{V}_b + \bar{V}_c = 0$, $\bar{V}_{ab} + \bar{V}_{bc} + \bar{V}_{ca} = 0$, $\bar{I}_a + \bar{I}_b + \bar{I}_c = 0$		

Balanced-only conversion: $Z_\Delta = 3 Z_Y$. Source and load must both be wye to solve a per-phase equivalent; a delta at either end is converted first.

WORKED EXAMPLE • MODULE 2, EXAMPLE 4 (P.60) • CORE • WORKED SOLUTION OURS; AGREES WITH THE SOLVED DECK, P.61

In a balanced three-phase circuit, $\bar{V}_{ab} = 173.2\angle 0^\circ \text{ V}$. Determine all the phase and line voltages and currents in a Y-connected load having $Z_L = 10\angle 20^\circ \Omega$.

Divide by $\sqrt{3}$ and rotate back by 30° :

$$\bar{V}_{an} = \frac{173.2}{\sqrt{3}} \angle (0^\circ - 30^\circ) = 100 \angle -30^\circ \text{ V},$$

$$\bar{V}_{bn} = 100 \angle -150^\circ, \quad \bar{V}_{cn} = 100 \angle 90^\circ \text{ V},$$

$$\bar{V}_{ab} = 173.2 \angle 0^\circ, \quad \bar{V}_{bc} = 173.2 \angle -120^\circ, \quad \bar{V}_{ca} = 173.2 \angle 120^\circ \text{ V}.$$

In a wye the line current is the phase current:

$$\bar{I}_a = \frac{100 \angle -30^\circ}{10 \angle 20^\circ} = 10 \angle -50^\circ \text{ A}, \quad \bar{I}_b = 10 \angle -170^\circ, \quad \bar{I}_c = 10 \angle 70^\circ \text{ A}.$$

The load is balanced, so $\bar{I}_n = 0$.

Three-phase power, and measuring it

ON THE FORMULA SHEET

$$P_{total} = 3 V_{ph} I_{ph} \cos \theta = \sqrt{3} V_L I_L \cos \theta = \frac{V_L^2}{Z_Y} \cos \theta,$$

$$Q_{total} = 3 V_{ph} I_{ph} \sin \theta = \sqrt{3} V_L I_L \sin \theta = \frac{V_L^2}{Z_Y} \sin \theta.$$

Star or delta alike; θ is the phase impedance angle.

The trap: $S_{total} = 3 \bar{V}_{ph} \bar{I}_{ph}^* \neq \sqrt{3} \bar{V}_L \bar{I}_L^*$ – the magnitudes agree but the angles do not, so build S_{total} from phase quantities.

A wattmeter reads $V I \cos \phi$: voltage coil across the terminals, current coil in series in the branch. Three-wattmeter (wye load, needs a neutral): total $P_A + P_B + P_C$. Two-wattmeter (delta load, no neutral): total $P_1 + P_2$.

Single-phase R, L and C

Every branch sees the same source voltage E , so currents add as phasors:

ON THE FORMULA SHEET

$$\bar{I}_R = \frac{E}{R} \angle 0^\circ, \quad \bar{I}_L = \frac{E}{X_L} \angle -90^\circ, \quad \bar{I}_C = \frac{E}{X_C} \angle +90^\circ, \quad X_L = 2\pi f L, \quad X_C = \frac{1}{2\pi f C},$$

$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C = I_R - j(I_L - I_C), \quad P = E I_R, \quad Q = E(I_L - I_C), \quad PF = \frac{I_R}{|\bar{I}|}.$$

WORKED EXAMPLE · 1-PH LAB MANUAL, TABLE 1.6 STATION 1 · CORE · PROBLEM AND SOLUTION OURS

Station 1: $R = 171.43 \Omega$, $L = 0.8 \text{ H}$, $C = 2.2 \mu\text{F}$, $E = 120 \text{ V}$ at 60 Hz , ideal elements. Predict I_1 (R), I_2 (L), I_3 (C), and the supply current, active power, reactive power and power factor for R, L, C, RL, RC and RLC. $X_L = 2\pi(60)(0.8) = 301.6 \Omega$ and $X_C = 1/(2\pi(60)(2.2 \times 10^{-6})) = 1205.7 \Omega$, so $I_R = 0.700 \text{ A}$, $I_L = 0.398 \text{ A}$ and $I_C = 0.0995 \text{ A}$.

TABLE 5 station 1, the base cases and RLC (ideal)

ON	A/M5	P (METER)	Q	S	PF
R	0.700 A	84.0 W (P1)	0	84.0 VA	1.000
L	0.398 A	0 (B/M11)	+47.7 VAR	47.7 VA	0 (lags 90°)
C	0.100 A	0 (B/M11)	-11.9 VAR	11.9 VA	0 (leads 90°)
RLC	0.761 A	84.0 W (P1)	+35.8 VAR	91.3 VA	0.920 lag

Practice, with answers

The Module 2 deck's own examples and the lab's transfer station; full solutions are in the lesson.

- (Module 2, Example 1, p.22)** A load on a 79.7 kV busbar is $Z = 80 + j60 \Omega$. Find the real and reactive power it absorbs and its power factor. *Answer:* $\bar{I} = 797 \angle -36.87^\circ \text{ A}$, $S = 63.52 \angle 36.87^\circ \text{ MVA}$, so $P = 50.82 \text{ MW}$, $Q = 38.11 \text{ MVAR}$, $PF = 0.8$ **lagging**.
- (Module 2, Example 7, p.66)** A wye source with a 440 V line voltage supplies load 1, 4Ω resistors in wye, and load 2, 6Ω resistors in delta. Find each load's phase voltage and line current, the source line current and the power delivered. *Answer:* $V_{ph,1} = 254.0 \text{ V}$, $I_{L,1} = 63.51 \text{ A}$; $V_{ph,2} = 440 \text{ V}$, $I_{ph,2} = 73.33 \text{ A}$, $I_{L,2} = 127.0 \text{ A}$; source $I_L = 190.5 \text{ A}$; $P = \sqrt{3} \times 440 \times 190.5 = 145.2 \text{ kW}$.
- (Lab transfer, station 14)** 1200Ω , 1.6 H and $4.4 \mu\text{F}$ at 120 V , 60 Hz . Predict A or M5 and the power factor for RL and RLC, and which ammeter reads most in RLC. *Answer:* RL 0.223 A , PF 0.45 lagging; RLC 0.100 A at unity PF, with I_2 and I_3 each reading 0.199 A , twice the supply current.