

ECE 231 course notes: technical material by topic

Semiconductor Physics and Devices, textbook Neamen 4th edition – the technical content of every lesson at <https://lessons.ihsan.cc/ece231/>, merged across the overview and lab lessons and organised by topic rather than by lesson. This lesson uses $kT = 0.025852$ eV at 300 K; the official problem-set solutions use 0.0259 eV, so some last digits differ.

Why a solid has a bandgap

Neamen 1 and 3.

Close-packed atoms split each level into N closely spaced levels forming a **band**; below a critical separation a **bandgap** E_g opens between bands. Silicon's gap is 1.12 eV, diamond's 5.5 eV – only silicon conducts at room temperature, since thermal excitation across E_g falls off exponentially with gap size. The vacancy left behind, the **hole**, is a positive carrier; thermal generation always makes carriers in pairs. Smaller lattice spacing means greater wavefunction overlap and a *wider* gap; dilating the lattice narrows it.

A carrier's response to a field is its **effective mass**:

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$$\frac{1}{m^*} = \frac{1}{\hbar^2} \frac{d^2 E}{dk^2}$$

Near a band edge $E(k) = C_1 k^2$ and $m^* = \hbar^2 / (2C_1)$: a sharply curved band gives a light carrier – why GaAs ($m_n^* = 0.067 m_0$, mobility about 8500 cm²/V·s) suits high-frequency devices where silicon ($m_n^* \approx 1.08 m_0$, about 1350 cm²/V·s) does not.

1.1 Worked example

PS1 -- Q5 · CORE · OFFICIAL SOLUTION AGREES

Two semiconductors have parabolic conduction-band minima at the origin. Curve A reaches $E = 0.05$ eV and curve B reaches $E = 0.5$ eV, both at $k = \pm 0.08 \text{ \AA}^{-1}$. Find m_A^* and m_B^* in units of m_0 .

WORKED SOLUTION

One point off each curve fixes C_1 . Converting units: $k = 0.08 \text{ \AA}^{-1} = 8.0 \times 10^8 \text{ m}^{-1}$, $k^2 = 6.4 \times 10^{17} \text{ m}^{-2}$, $\hbar = 1.054 \times 10^{-34} \text{ J}\cdot\text{s}$. Curve A at $E = 0.05$ eV:

$$C_{1,A} = \frac{0.05 \times 1.60 \times 10^{-19}}{6.4 \times 10^{17}} = 1.25 \times 10^{-38} \text{ J} \cdot \text{m}^2,$$

$$m_A^* = \frac{\hbar^2}{2C_{1,A}} = \frac{1.112 \times 10^{-68}}{2.50 \times 10^{-38}} = 4.45 \times 10^{-31} \text{ kg} = 0.49 m_0.$$

Curve B reaches ten times the energy at the same k , so C_1 is ten times larger and the mass ten times smaller: $m_B^* = 4.45 \times 10^{-32} \text{ kg} = 0.049 m_0$.

Answer: $m_A^* = 0.49 m_0$, $m_B^* = 0.049 m_0$.

1.2 Short practice

PS1 -- Q2 · INTRO · OFFICIAL SOLUTION AGREES

$2 \times 10^{16} \text{ cm}^{-3}$ boron atoms are added to silicon as a substitutional impurity, uniformly distributed. What is the distance between boron atoms?

ANSWER

Modelling the volume per dopant, $1/N_B$, as a cube of edge d : $d = (1/2 \times 10^{16})^{1/3} = 3.68 \times 10^{-6} \text{ cm} \approx 37 \text{ nm}$ – about 136 silicon lattice spacings, one boron per 2.5 million silicon atoms.

How many carriers, and how doping sets the number

Neamen 4.

Carrier density is **density of states times Fermi function**:

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$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

integrated over energy; in the Maxwell-Boltzmann tail ($E_c - E_F \gg kT$) that collapses to:

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$$n_0 = N_c \exp\left(\frac{-(E_c - E_F)}{kT}\right), \quad p_0 = N_v \exp\left(\frac{-(E_F - E_v)}{kT}\right)$$

where N_c, N_v are the effective densities of states (material and temperature only, each scaling as $T^{3/2}$); how the sample was made enters only through E_F . Multiplying the two cancels E_F : $n_0 p_0 = N_c N_v \exp(-E_g/kT) = n_i^2$, the **mass-action law** – fixed by material and temperature alone, never by doping. The **intrinsic Fermi level** sits off midgap by the carriers' mass asymmetry:

$$E_{Fi} - E_{\text{midgap}} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right), \quad p_0 = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right)$$

Doping moves E_F , not E_g . Donors (group V) sit ~ 0.03 eV below E_c , acceptors (group III) ~ 0.05 eV above E_v , both essentially all ionised at 300 K. With both present only the difference counts (the compensated formula $n_0 = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$ covers every regime), and when $N_d - N_a \gg n_i$ simply $n_0 = N_d - N_a$. Every kT of Fermi-level movement is a factor of e in n_0 , since $n_0 \propto \exp(-(E_c - E_F)/kT)$.

2.1 Worked example

PS1 -- Q11 · CORE · OFFICIAL SOLUTION AGREES

The electron concentration in silicon at $T = 300$ K is $n_0 = 2 \times 10^5 \text{ cm}^{-3}$. (a) Position of the Fermi level relative to E_v . (b) Find p_0 . (c) n-type or p-type?

WORKED SOLUTION

(a)

$$E_c - E_F = kT \ln\left(\frac{N_c}{n_0}\right) = (0.025852) \ln(1.4 \times 10^{14}) = 0.842 \text{ eV},$$

so $E_F - E_v = E_g - 0.842 = 1.12 - 0.842 = 0.278 \text{ eV}$.

(b) $p_0 = n_i^2/n_0 = (1 \times 10^{10})^2/2 \times 10^5 = 5 \times 10^{14} \text{ cm}^{-3}$.

(c) Holes outnumber electrons by 2.5×10^9 and E_F sits well below midgap: **p-type**.

Answers: (a) 0.278 eV above E_v · (b) $5 \times 10^{14} \text{ cm}^{-3}$ · (c) p-type.

Official solution: $E_c - E_F = 0.8436 \text{ eV}$, $E_F - E_v = 0.2764 \text{ eV}$, $p_0 = 2.414 \times 10^{14} \text{ cm}^{-3}$, p-type – from the hole side, $p_0 = N_v \exp(-(E_F - E_v)/kT)$, and $kT = 0.0259 \text{ eV}$.

2.2 Short practice

PS2 -- Q3 · CORE · AI-WORKED

Silicon at 300 K is doped with $N_d = 6 \times 10^{15} \text{ cm}^{-3}$, all ionised. (a) Find $E_c - E_F$. (b) How many more donors must be added to move E_F a distance kT closer to E_c ?

ANSWER

(a) $n_0 = N_d$ (the n_i^2/N_d correction is 1.7×10^4 , negligible): $E_c - E_F = 0.025852 \ln(2.8 \times 10^{19}/6 \times 10^{15}) = 0.2184 \text{ eV}$. (b) A kT shift needs $N'_d = eN_d = 1.63 \times 10^{16} \text{ cm}^{-3}$, an addition of $1.03 \times 10^{16} \text{ cm}^{-3}$.

What moves carriers, and what happens away from equilibrium

Neamen 5 and 6.

Every current is **drift**, driven by a field, or **diffusion**, driven by a concentration gradient, or both:

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$$v_{dn} = -\mu_n \mathcal{E}, \quad \mu_n = \frac{e\tau_n}{m_n^*}, \quad J_{drift} = e(\mu_n n + \mu_p p)\mathcal{E} = \sigma \mathcal{E}, \quad \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}$$

The **Einstein relation** links mobility and diffusion via the same scattering: phonons worsen mobility with temperature ($\mu \propto T^{-3/2}$), ionized impurities improve it ($\mu \propto T^{3/2}/N_I$), and mobility peaks where the two cross. Away from equilibrium each carrier gets its own **quasi-Fermi level**; the **continuity equation** plus a recombination **lifetime** tracks how the excess population decays.

Why a junction passes current one way

Neamen 7 and 8.

Diffusion uncovers fixed dopant charge, building a field that cancels it at equilibrium (the **depletion approximation**: no mobile carriers inside, neutral outside; $x_p N_a = x_n N_d$ puts most of the width on the lightly doped side):

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$$V_{bi} = \frac{kT}{e} \ln \left(\frac{N_a N_d}{n_i^2} \right), \quad |\mathcal{E}_{max}| = \frac{2(V_{bi} - V_a)}{W}$$

The minority-carrier diffusion equation gives the **ideal diode equation**; forward bias raises injected current exponentially, reverse bias saturates it at $-J_s$:

$$J = J_s \left(\exp \left(\frac{eV_a}{kT} \right) - 1 \right)$$

Controlling a channel you never touch

Neamen 9 and 10.

Work-function mismatch at a metal-semiconductor contact leaves a barrier: a rectifying **Schottky contact**, or an **ohmic contact** if the surface is doped heavily enough to tunnel through it. Over an oxide, no steady gate current flows; sweeping the gate on p-type gives **accumulation**, **depletion**, then **inversion** (an n-type surface layer) past the **threshold voltage** V_T .

With n-type source/drain either side of the gate, the inversion layer connects them above V_T : a gate-controlled resistor (**triode**) at small V_{DS} , until $V_{DS} = V_{GS} - V_T$ pinches it off at the drain and the current flattens (**saturation**).

Current gain, and junctions that trade with light

Neamen 12 and 14.

In an npn BJT, forward bias on the emitter-base junction injects electrons into the base; the base is thinner than a diffusion length, so most cross before recombining and reach the collector. Two fractions close to one – **base transport factor** and **emitter injection efficiency** – multiply to give α , and $\beta = \alpha / (1 - \alpha)$: at $\alpha = 0.99$, $\beta = 99$; at $\alpha = 0.995$, $\beta = 199$. A thin,

lightly doped base raises the transport factor; a heavily doped emitter against the lighter base raises the injection efficiency.

Run the same junction with photons: absorption in the depletion region creates a pair the field separates – **photodetector** (reverse biased) or **solar cell** (unbiased). Forward biased, carriers recombine; a **direct-gap** material like GaAs emits light efficiently (**LED**), silicon's indirect gap makes it a poor emitter.

6.1 Check yourself

Why is the base made thin and lightly doped, and the emitter heavily doped?

ANSWER

A thin base raises the base transport factor (carriers cross before recombining); a heavily doped emitter against a lightly doped base raises the injection efficiency (negligible back-injection). Both push α toward one, and $\beta = \alpha / (1 - \alpha)$ converts that into large gain.

Lab instrumentation and measurement

Instrument and measurement principles from Lab 1 – not lab procedure or logistics.

Resistors. A four-band resistor's value and tolerance band:

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$$R = (10 d_1 + d_2) \times 10^m \Omega, \quad R_{\min} = R(1 - \frac{t}{100}), \quad R_{\max} = R(1 + \frac{t}{100})$$

Digit colours: black 0, brown 1, red 2, orange 3, yellow 4, green 5, blue 6, violet 7, grey 8, white 9; the same colours as a multiplier give that power of ten, gold 0.1, silver 0.01; tolerance brown 1%, red 2%, gold 5%, silver 10%. A decade box gives the same nominal-vs-actual gap: one dial per decade, each step its own resistor with its own tolerance, and a box set to zero is not zero ohms – measure it at the setting used.

7.1 Worked example

Yellow, violet, red, gold: digits 4 and 7, multiplier 10^2 , so $R = 4700 \Omega$; gold promises the real part is within 5% of that.

ANSWER

$$R_{\min} = 4700(0.95) = 4465 \, \Omega, R_{\max} = 4700(1.05) = 4935 \, \Omega.$$

Measuring resistance. In 2-wire mode the lead resistance sits inside what is sensed; in 4-wire mode a separate sense pair carries no current, so lead resistance drops nothing.

ANSWER

A $10 \, \Omega$ part with $0.3 \, \Omega$ in each lead reads $10.6 \, \Omega$ 2-wire (6% high) and $10 \, \Omega$ 4-wire. Rule: 4-wire for small resistances, 2-wire for everything else.

DC supply. Each output holds either the set voltage (CV) or the current limit (CC), whichever the load makes true:

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$$R_{\text{load}} > \frac{V_{\text{set}}}{I_{\text{limit}}} \Rightarrow \text{CV}, I = \frac{V_{\text{set}}}{R_{\text{load}}}; \quad R_{\text{load}} < \frac{V_{\text{set}}}{I_{\text{limit}}} \Rightarrow \text{CC}, V = I_{\text{limit}} R_{\text{load}}$$

7.2 Short practice

5 V across a $470 \, \Omega$ resistor, current limit 50 mA. What does the ammeter read, and what mode is the supply in? Now replace the resistor with $10 \, \Omega$.

ANSWER

$470 \, \Omega > 5\text{V}/50\text{mA} = 100 \, \Omega$: CV, $I = 5/470 = 10.6 \, \text{mA}$. With $10 \, \Omega < 100 \, \Omega$: CC, $I = 50 \, \text{mA}$ and $V_R = 50\text{mA} \times 10 \, \Omega = 0.5 \, \text{V}$.

Meter placement and range. A voltmeter goes in parallel (nearly open, $10 \, \text{M}\Omega$ or more), an ammeter in series (nearly a short); a current function placed across a source is a short across the source. A range reads to 120% of its face value and shows OVLD above that; the last digit is about a millionth of the range, so the smallest range the reading fits in gives the most digits.

Sinusoids. The generator makes

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$$v(t) = V_{\text{off}} + V_p \sin(2\pi ft), \quad V_{pp} = 2V_p, \quad T = \frac{1}{f}, \quad V_{\text{rms}} = \sqrt{V_{\text{off}}^2 + \frac{V_p^2}{2}}$$

and for no offset $V_{\text{rms}} = V_p / \sqrt{2} \approx 0.707 V_p$. On the scope, a screen 10×8 divisions gives peak height $V_p / (\text{V/div})$ divisions, one period $T / (\text{s/div})$ divisions wide, and $10 (\text{s/div}) f$ cycles on screen.

7.3 Worked example

A 2 V peak, 1 kHz sine with no offset, read at 1 V/div and 500 $\mu\text{s}/\text{div}$.

ANSWER

$V_{pp} = 4 \text{ V}$, $V_{\text{rms}} = 1.41 \text{ V}$, $T = 1 \text{ ms}$; on screen, 2 divisions tall and 5 cycles wide.

Generator loading. The generator has a real 50Ω in series with its output; its display reports the voltage that would appear across the load it was told to expect. Set to 50Ω and driving a true 50Ω load, the display is correct; driving a $1 \text{ M}\Omega$ input instead (which halves nothing) delivers twice the displayed amplitude. Setting the load to High Z fixes the display for a high-impedance load such as a scope.