

Chapter 1: signals and systems

The printable for <https://lessons.ihsan.cc/ece207/ece207-ch1-signals-systems/> – key ideas, formulas and one worked example per topic, for reading away from the screen.

Six topics, Chapter 1: transformations, energy and power, even/odd parts, system properties, LTI response by shifted copies, network models with $D = d/dt$. The impulse and sifting property are Chapter 2 and not covered here.

Transformations of the time variable

Every transformation changes the clock, not the signal: the value at time t in the new signal is a value x already had, at some other time.

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Inversion: $y(t) \triangleq x(-t)$, Scaling ($a > 0$): $y(t) \triangleq x(at)$, Shift: $y(t) \triangleq x(t - t_0)$.

Scaling with $a = 2$ compresses toward the origin, $a = 1/2$ stretches; a shift by $t_0 > 0$ moves the picture right. In discrete time a and k_0 must be integers.

WORKED FOR THIS LESSON • MEDIUM • AI, FROM THE CHAPTER 1 NOTES

The Example 1.1.28 triangle lives on $0 \leq t < 3$ with its peak at $t = 2$. Where is $y(t) = x(-2t + 4)$ nonzero, where is its peak, and which side is steep?

DERIVATION • SKIP ON RE-READ

Nonzero where $0 \leq -2t + 4 < 3$. Subtract 4 and divide by -2 , which flips both inequalities: $1/2 < t \leq 2$.

The peak is where $-2t + 4 = 2$, so at $t = 1$. The rising side of x (argument 0 to 2) maps to t from 2 down to 1, a length of 1. The falling side (argument 2 to 3) maps to t from 1 down to $1/2$, a length of $1/2$. The steep side is on the left.

Energy and power

Two ways to measure the size of a signal, and why a signal has at most one of them.

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$$E \triangleq \int_{-\infty}^{\infty} |x(t)|^2 dt, \quad P \triangleq \lim_{a \rightarrow \infty} \frac{1}{2a} \int_{-a}^a |x(t)|^2 dt, \quad P = \frac{1}{T} \int_0^T |x(t)|^2 dt \text{ (periodic)}.$$

E finite $\Rightarrow$ *energy* signal; $0 < P < \infty \Rightarrow$ *power* signal. If E is finite then $P = 0$; if P is finite and nonzero then $E = \infty$ – a signal is never both.

Even and odd parts

Every signal is an even signal plus an odd one, and there is a formula for each.

x is even when $x(-t) = x(t)$, odd when $x(-t) = -x(t)$; an odd signal passes through zero, since $x(0) = -x(0)$. Any signal splits as:

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$$x_e(t) \triangleq \frac{1}{2} [x(t) + x(-t)], \quad x_o(t) \triangleq \frac{1}{2} [x(t) - x(-t)].$$

System properties

Linear, time invariant, memoryless, causal: what each one asks, and how to test it.

Linear. Response to $a_1x_1 + a_2x_2$ is $a_1y_1 + a_2y_2$; a zero input must give a zero output. **Time invariant** (Def 1.2.10):

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$$x_2(t) \triangleq x_1(t - t_0) \implies y_2(t) = y_1(t - t_0), \quad \text{for every } t_0.$$

Any t or k appearing *outside* the input's argument breaks time invariance. **Memoryless:** $y(t_0)$ depends only on $x(t_0)$, not on $x(t)$ at any other t . **Causal:** if two inputs agree for all $t \leq t_0$, their outputs must agree at t_0 .

TABLE 1 *four systems against all four properties*

SYSTEM	LINEAR	TI	MEMORYLESS	CAUSAL
$y = 1 + x$ (Ex 1.2.8)	no	yes	yes	yes
RC circuit (Ex 1.2.9, 1.2.14, 1.2.18)	yes	yes	no	yes
$y = x^2$ (Ex 1.2.11)	no	yes	yes	yes
$y[k] = k x[k]$ (Ex 1.2.12)	yes	no	yes	yes
$y[k] = \sum_{j=0}^4 x[k+j]$ (Ex 1.2.19)	yes	yes	no	no

WORKED FOR THIS LESSON · MEDIUM · AI, FROM THE CHAPTER 1 NOTES

Classify $y(t) = x(t-1)$, $y(t) = t x(t)$ and $y(t) = x(2t)$ on all four properties.

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$x(t-1)$: linear, time invariant, causal (it looks one second back), not memoryless.

$t x(t)$: linear, memoryless, causal, but time varying: $y_2(t) = t x_1(t-t_0)$ while $y_1(t-t_0) = (t-t_0) x_1(t-t_0)$.

$x(2t)$: linear, but time varying ($y_2(t) = x_1(2t-t_0)$ while $y_1(t-t_0) = x_1(2t-2t_0)$), not memoryless, and not causal: $y(1) = x(2)$ reads the future.

LTI response by shifted copies

Know the response to one pulse, and you know the response to any staircase built from it.

If y_0 is the response to x_0 , then by linearity and time invariance:

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$$x(t) = \sum_i c_i x_0(t-t_i) \implies y(t) = \sum_i c_i y_0(t-t_i).$$

Write x as scaled, delayed copies of x_0 ; the output side is copy-and-paste.

TUTORIAL 1, Q7 · MEDIUM · OFFICIAL

An LTI system responds to the unit pulse $x_0(t)$ (1 on $(0, 1]$, 0 elsewhere) with $y_0(t)$. Find its response to the input that is 1 on $(0, 1]$, 3 on $(1, 2]$, 0 on $(2, 3]$, -2 on $(3, 4]$ and 0 elsewhere.

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Each unit interval is a copy of x_0 delayed to start there, scaled by the height; the gap on $(2, 3]$ has coefficient 0, so it does not appear:

$$x(t) = x_0(t) + 3x_0(t-1) - 2x_0(t-3).$$

Because the system is LTI,

$$y(t) = y_0(t) + 3y_0(t-1) - 2y_0(t-3).$$

Network models with $D = d/dt$

Turn a circuit into one differential equation relating input and output.

A series RLC circuit driven by $x(t)$ with loop current $y(t)$: KVL adds the three element voltages, then differentiating once removes the integral:

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$$L \frac{d^2 y}{dt^2} + R \frac{dy}{dt} + \frac{1}{C} y(t) = \frac{dx}{dt}.$$

With more than one loop, write $D \triangleq d/dt$, treat it as a number, eliminate currents by Cramer's rule, then read each power of D back as a derivative.

EXAMPLE 1.2.22 · MEDIUM · OFFICIAL

Loop 1 has a 2Ω resistor and a 2 H inductor, loop 2 has a 4Ω resistor and a 1 H inductor, sharing a 3Ω resistor carrying $y_1 - y_2$. Input $x(t)$ drives loop 1; output $y = y_1$. Find the differential equation.

DERIVATION · SKIP ON RE-READ

$$(2D + 5)y_1 - 3y_2 = x(t), \quad -3y_1 + (D + 7)y_2 = 0.$$

Cramer's rule for y_1 :

$$y_1 = \frac{(D + 7)x}{(2D + 5)(D + 7) - 9} = \frac{(D + 7)x}{2D^2 + 19D + 26}.$$

Read the powers of D back as derivatives:

$$2 \frac{d^2 y}{dt^2} + 19 \frac{dy}{dt} + 26 y(t) = \frac{dx}{dt} + 7 x(t).$$

Practice, with answers

Numbers unchanged from the lesson; full steps are in the lesson if you want them.

1. **(Tutorial 1, Q1(b), easy, official)** Find the energy of $x(t) = t[u(t) - u(t - 5)]$.
Answer: $E = \int_0^5 t^2 dt = 125/3$.
2. **(Worked for this lesson, easy, ai, from Tutorial 1)** Classify $x(t) = e^{-2t}u(t)$ and $x(t) = u(t)$ as energy or power signals, and give the number.
Answer: $e^{-2t}u(t)$ is an energy signal, $E = 1/4$ (power 0); $u(t)$ is a power signal, $P = 1/2$ (energy infinite).
3. **(Worked for this lesson, medium, ai, from the Chapter 1 notes)** Find the even and odd parts of $x(t) = e^{-t}u(t)$ for $t \neq 0$.
Answer: $x_e(t) = \frac{1}{2}e^{-|t|}$, $x_o(t) = \frac{1}{2}\text{sgn}(t)e^{-|t|}$.
4. **(Worked for this lesson, easy, ai, from Tutorial 1)** Same LTI system as §5. The input is 2 on $(0, 2]$, -1 on $(2, 3]$ and 0 elsewhere. Write the output.
Answer: $x(t) = 2x_0(t) + 2x_0(t - 1) - x_0(t - 2)$, so $y(t) = 2y_0(t) + 2y_0(t - 1) - y_0(t - 2)$.
5. **(Worked for this lesson, medium, ai, from the Chapter 1 notes)** Change the 4Ω resistor in loop 2 of Example 1.2.22 to 1Ω , keep everything else, and find the differential equation for $y = y_1$.
Answer: $-3y_1 + (D+4)y_2 = 0$ gives $y_1 = (D+4)x/(2D^2+13D+11)$, so $2y'' + 13y' + 11y = x' + 4x$.