

Course overview: vector calculus to residues

Key-points printable for <https://lessons.ihsan.cc/ece206/ece206-course-overview/> – the course facts, the formula sheet and the highest-value worked examples.

The course

Weights, meetings, and every date the Fall 2026 outline fixes.

ECE206 (cross-listed with MATH 212) is vector calculus in weeks 1–7, closing on Maxwell's equations, then complex analysis in weeks 9–13. Lectures by Sean Speziale (office hours Wed 11:30–12:20, Fri 10:30–11:30, MC 6512); LEARN, Mobius and Piazza.

WHEN AND WHAT IS GRADED

COMPONENT	WEIGHT	WHEN
Lecture (001) / Tutorial (101)	–	Mon/Fri 4:00–5:20 p.m. / Tue 4:30–5:20 p.m., PSE 4043
Mobius assignments (optional)	0 or 10%	most Mondays, 11:59 p.m.
Quizzes, best 4 of 5	20%	Tue tutorial
Midterm	25%	Fri Oct 30, 4:00–5:30 p.m.
Final exam	45 or 55%	date not announced

Mobius is graded both ways and the better kept: Mobius 0% + final 55%, or Mobius 10% + final 45%, so doing the assignments can only help. The final falls in the Dec 10–23 exam period; each section below names its quiz.

Curves and surfaces

Weeks 1 to 2; Quiz 1 (Sept 22) covered parametric curves and surfaces.

A curve is a map from an interval, $\vec{g}(t) = (x(t), y(t), z(t))$; a surface is a map from a plane region, $\vec{g}(u, v)$. Match a sum of two squares in the implicit equation to $\sin^2 + \cos^2 = 1$ to read off a parameterisation; a linear constraint is simply solved for the remaining coordinate.

ON THE FORMULA SHEET

$$\vec{v}(t) = \vec{g}'(t), \quad \text{speed} = \|\vec{g}'(t)\|, \quad s = \int_a^b \|\vec{g}'(t)\| dt.$$

$\vec{v}(t)$ is the velocity (tangent to the curve); its magnitude is the speed; arc length s integrates speed over the parameter interval.

ON THE FORMULA SHEET

$$\vec{n} = \vec{g}_u \times \vec{g}_v, \quad S = \iint_D \|\vec{g}_u \times \vec{g}_v\| du dv.$$

$\vec{n}$ is normal to the surface (order of the cross product fixes orientation); S integrates the area of the parallelogram spanned by the two tangents.

Worked example (Problem Set 1)

ECE 206 PROBLEM SET 1, Q5 · CORE · OFFICIAL

Find the length of the curve $\vec{g}(t) = t^3 \hat{i} + t^2 \hat{j} + t^3 \hat{k}$ for $0 \leq t \leq 1$.

DERIVATION · SKIP ON RE-READ

$\vec{g}'(t) = 3t^2 \hat{i} + 2t \hat{j} + 3t^2 \hat{k}$, magnitude $\sqrt{18t^4 + 4t^2} = t\sqrt{18t^2 + 4}$ (legitimate since $t \geq 0$).

$$s = \int_0^1 t\sqrt{18t^2 + 4} dt.$$

Substitute $u = 18t^2 + 4$, $du = 36t dt$, limits 4 to 22.

$$s = \frac{1}{36} \int_4^{22} \sqrt{u} du = \frac{1}{54} (22^{3/2} - 8).$$

Practice (answers only)

- **PS1 Q1.** $\vec{r} = (2, 3, -1)$, $\vec{s} = (1, -2, 3)$: angle $\theta = 2\pi/3$; parallelogram area $7\sqrt{3}$.
- **Lecture 3, p. 12.** Surface area of the cone $\vec{g}(u, v) = (u \cos v, u \sin v, u)$, $0 \leq u \leq 1$, $0 \leq v \leq 2\pi$: $S = \sqrt{2}\pi$.
- **PS2 Q4.** Sphere $x^2 + y^2 + z^2 = 9$ between $z = 1$ and $z = 2$: annulus $\sqrt{5} \leq r \leq \sqrt{8}$ in D_{xy} , $S = 6\pi$ (matches Archimedes' zone formula $2\pi Rh$).

Multiple integrals

Week 3, orientation only – full treatment is a separate lesson, *Multiple integrals*, at <https://lessons.ihsan.cc/ece206/multiple-integrals/>.

ON THE FORMULA SHEET

$$dA_{\text{polar}} = r \, dr \, d\theta, \quad dV_{\text{cyl}} = r \, dr \, d\theta \, dz, \quad dV_{\text{sph}} = \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta.$$

Each factor is the absolute value of that coordinate map's Jacobian. Quiz 2 (Oct 6) covers this plus line integrals.

Line integrals and Green's theorem

Weeks 4 to 5.

ON THE FORMULA SHEET

$$\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{g}(t)) \cdot \vec{g}'(t) \, dt, \quad \int_C \nabla \phi \cdot d\vec{r} = \phi(\vec{g}(b)) - \phi(\vec{g}(a)).$$

The first is the work done by $\vec{F}$ along C ; the second says a gradient field's line integral depends only on the endpoints (conservative). Test: $P_y = Q_x$ (mixed partials of the potential agree) $\iff \nabla \times \vec{F} = \vec{0}$ on a simply connected region.

ON THE FORMULA SHEET

$$\oint_C P \, dx + Q \, dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA.$$

Green's theorem: circulation around a positively oriented boundary C equals a double integral of local rotation over the region D it encloses. Internal edges cancel in pairs; only the outer boundary survives.

Flux, divergence, curl and Maxwell

Weeks 5 and 7. Divergence is flux per unit volume, curl is circulation per unit area.

ON THE FORMULA SHEET

$$\iint_{\Sigma} \vec{F} \cdot d\vec{S} = \iint_D \vec{F}(\vec{g}(u, v)) \cdot (\vec{g}_u \times \vec{g}_v) du dv, \quad \nabla \cdot \vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}.$$

Flux integrates the normal component over the surface; divergence is a source density.

ON THE FORMULA SHEET

$$\iiint_V \vec{F} \cdot d\vec{S} = \iiint_V (\nabla \cdot \vec{F}) dV, \quad \nabla \times \vec{F} = (R_y - Q_z, P_z - R_x, Q_x - P_y).$$

Divergence Theorem: net flux out of a closed surface equals the divergence integrated over the enclosed volume.

ON THE FORMULA SHEET

$$\oint_{\partial\Sigma} \vec{F} \cdot d\vec{r} = \iint_{\Sigma} (\nabla \times \vec{F}) \cdot d\vec{S}.$$

Stokes' theorem: circulation around the boundary of a surface equals the flux of the curl through it; Green's theorem is the flat special case. Identities: $\nabla \times \nabla\phi = \vec{0}$ (a gradient has no rotation) and $\nabla \cdot (\nabla \times \vec{F}) = 0$ (a curl has no sources).

MAXWELL'S EQUATIONS, DIFFERENTIAL FORM

DIFFERENTIAL FORM	SAYS
$\nabla \cdot \vec{D} = \rho$	electric flux out of a closed surface counts the charge inside
$\nabla \cdot \vec{B} = 0$	no magnetic sources – flux through any closed surface is zero
$\nabla \times \vec{E} = -\partial\vec{B}/\partial t$	changing magnetic flux drives circulation of $\vec{E}$
$\nabla \times \vec{H} = \vec{J} + \partial\vec{D}/\partial t$	current and changing electric flux drive circulation of $\vec{H}$

The divergence rows are the Divergence Theorem; the curl rows are Stokes. Quiz 3 (Oct 20) covers this; the midterm (Oct 30) closes the examinable first half.

Complex functions

Weeks 9 to 10.

ON THE FORMULA SHEET

$$f'(z) = \lim_{h \rightarrow 0} \frac{f(z+h) - f(z)}{h}, \quad h \in \mathbb{C}, \quad u_x = v_y, \quad u_y = -v_x.$$

The limit must agree from every direction h , which gives the Cauchy-Riemann equations. Differentiable on an open set means analytic; u, v are then harmonic conjugates, and where $f'(z) \neq 0$ the map is conformal (angle-preserving). Quiz 4 (Nov 10) covers this.

Contours and residues

Weeks 11 to 13.

ON THE FORMULA SHEET

$$\int_C f(z) dz = \int_a^b f(z(t)) z'(t) dt, \quad f(z_0) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z - z_0} dz.$$

Cauchy-Goursat: the integral is zero for f analytic on and inside a simple closed contour, so a contour may be shrunk onto small circles around the singularities. Cauchy's integral formula recovers $f(z_0)$ from its boundary values.

ON THE FORMULA SHEET

$$f(z) = \sum_{n=-\infty}^{\infty} a_n (z - z_0)^n, \quad \text{Res}(f, z_0) = a_{-1}, \quad \oint_C f(z) dz = 2\pi i \sum_k \text{Res}(f, z_k).$$

The Laurent series's negative part classifies the singularity (none: removable; finitely many, to order m : pole of order m ; infinitely many: essential). Only the a_{-1} term survives a loop, so a contour integral is $2\pi i$ times the enclosed residues. Simple pole: $\text{Res} = \lim_{z \rightarrow z_0} (z - z_0) f(z)$. Quiz 5 (Nov 24) covers this; the final exam covers the whole course.