

Course overview

The key points of <https://lessons.ihsan.cc/ece203/01-course-overview/>: the course page, the formulas from each block of notes, one worked example per block and practice with answers.

The course

Figures quoted from the Fall 2026 outline. ECE 203 is probability theory (statistics is ECE 307). Taught by Patrick Mitran from 39 numbered lecture notes following Ross's *A First Course in Probability* (10th ed.); the notes, not the slides, are the course.

The grade: **problem sets** 20% (nine, lowest dropped; most Fridays 11:59pm, Crowdmark), the **midterm** 30% (Tue 27 Oct 2026, 4:30–6:00pm) and the **final exam** 50% (TBD, university final exam schedule).

Late submissions lose **1% per minute**; generative AI is **not permitted** on graded work.

Axioms, sample spaces and counting

Notes 1–4, Ross ch. 1–2. A **sample space** S is every outcome an experiment could produce; an **event** is a subset $E \subset S$. P is any assignment satisfying three axioms:

$$(A1) \ 0 \leq P[E] \leq 1 \quad (A2) \ P[S] = 1 \quad (A3) \ P\left[\bigcup_i E_i\right] = \sum_i P[E_i] \text{ for disjoint } E_i$$

For overlapping events $P[E \cup F] = P[E] + P[F] - P[EF]$, where EF is shorthand for $E \cap F$. When S is finite and every outcome is equally likely (an extra modelling assumption, not something the axioms supply), $P[E] = |E|/|S|$: probability becomes counting.

WORKED EXAMPLE · ECE203 TUTORIAL 1 (2021), EXAMPLE T1.4 · CORE · OFFICIAL

An urn has 10 balls numbered 1 to 10. Three are drawn at random. What is the probability that one is even and two are odd?

With order: $|S_1| = 10 \times 9 \times 8 = 720$; the even ball can be first, second or third, each in $5 \times 5 \times 4 = 100$ ways, so $300/720 = 5/12$. Without order: $|S_2| = \binom{10}{3} = 120$ and

$|E_2| = \binom{5}{2} \binom{5}{1} = 50$, so $50/120 = 5/12$ again. Count S and E the same way and the factor of 6 cancels.

Conditioning, Bayes and independence

Notes 5–7, Ross ch. 3. Conditioning renormalises to what you have learned, and splitting S into a partition $F_1, \dots, F_n$ gives the **law of total probability**:

$$P[E | F] = \frac{P[EF]}{P[F]} \quad (P[F] > 0), \quad P[E] = \sum_{i=1}^n P[E | F_i] P[F_i]$$

Bayes' formula runs it backward, from effect E to cause F_j :

$$P[F_j | E] = \frac{P[E | F_j] P[F_j]}{\sum_i P[E | F_i] P[F_i]}$$

$P[F_j]$ is the **prior**, $P[F_j | E]$ the **posterior**. Events are **independent** when $P[EF] = P[E]P[F]$, equivalently $P[E | F] = P[E]$; for three or more events the product rule must hold over every subset, not just the pairs.

WORKED EXAMPLE · ECE203 TUTORIAL 2 (2021), EXAMPLE T2.2 · CORE · OFFICIAL

A pouch holds a four-sided and a six-sided die; you pick one at random and roll it. (a) What is the probability of rolling i , $i = 1, \dots, 6$? (b) Given a 3, what is the probability you picked the six-sided die?

Let A be the four-sided die, $B = A^c$ the six-sided one (each $1/2$), E_i a roll of i . (a) Total probability: for $i \leq 4$, $\frac{1}{4} \cdot \frac{1}{2} + \frac{1}{6} \cdot \frac{1}{2} = 5/24$; for $i = 5, 6$, $0 \cdot \frac{1}{2} + \frac{1}{6} \cdot \frac{1}{2} = 2/24$. (b) Bayes:

$$P[B | E_3] = \frac{\frac{1}{6} \cdot \frac{1}{2}}{\frac{1}{6} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{2}} = \frac{2}{5}.$$

Discrete random variables

Notes 8–14, Ross ch. 4. A **discrete** random variable X has **pmf** $p_X(a) = P[X = a]$. Its **expected value** and **LOTUS** are weighted averages against p_X (in general $E[g(X)] \neq g(E[X])$); **variance** measures spread:

$$E[X] = \sum_x x p_X(x), \quad E[g(X)] = \sum_x g(x) p_X(x), \quad \text{Var}[X] = E[X^2] - (E[X])^2$$

and $E[aX + b] = a E[X] + b$, $\text{Var}[aX + b] = a^2 \text{Var}[X]$. Most problems reduce to recognising one of four families:

TABLE 1 *the four discrete families*

FAMILY	COUNTS	PMF	$E[X]$	$\text{Var}[X]$
Bernoulli(p)	one trial: success?	$p, 1 - p$	p	$p(1 - p)$
Binomial(n, p)	successes in n trials	$\binom{n}{k} p^k (1 - p)^{n-k}$	np	$np(1 - p)$
Poisson(λ)	rare events in a window	$\frac{\lambda^k}{k!} e^{-\lambda}$	λ	λ
Geometric(p)	trials to first success	$(1 - p)^{k-1} p$	$1/p$	$(1 - p)/p^2$

The Poisson is the limit of a binomial as $n \rightarrow \infty, p \rightarrow 0$ with $np = \lambda$ fixed.

WORKED EXAMPLE · ECE203 TUTORIAL 4 (2021), EXAMPLE T4.2 · CORE · OFFICIAL

N , the number of calls at a call centre in t seconds, is Poisson with $\lambda = \alpha t$, $\alpha = 0.2$ calls per second. (i) What is the probability of more than 3 calls in 10 seconds? (ii) Of exactly 12 calls in one minute?

(i) $\lambda = 0.2 \times 10 = 2$, so $P[N > 3] = 1 - e^{-2}(1 + 2 + \frac{2^2}{2} + \frac{2^3}{6}) = 1 - \frac{19}{3}e^{-2}$. (ii) $\lambda = 0.2 \times 60 = 12$, so $P[N = 12] = \frac{12^{12}}{12!} e^{-12} \approx 0.11437$.

Continuous random variables

Notes 15–21, Ross ch. 5. A random variable is **continuous** when a density f_X gives area as probability, so $P[X = a] = 0$; standardising turns any normal into the standard normal, so one table answers every Gaussian question:

$$P[X \in B] = \int_B f_X(x) dx, \quad \int_{-\infty}^{\infty} f_X dx = 1; \quad Z = \frac{X - \mu}{\sigma} \sim N(0, 1), \quad \Phi(z) = P[Z \leq z]$$

and $Q(z) = 1 - \Phi(z)$. The exponential is **memoryless**: $P[X > s + t \mid X > t] = P[X > s]$.

TABLE 2 *the three continuous families*

FAMILY	PDF	$E[X]$	$\text{Var}[X]$
Uniform(a, b)	$1/(b - a)$ on (a, b)	$(a + b)/2$	$(b - a)^2/12$
Normal $N(\mu, \sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/(2\sigma^2)}$	μ	σ^2
Exponential(λ)	$\lambda e^{-\lambda x}$ on $x \geq 0$	$1/\lambda$	$1/\lambda^2$

WORKED EXAMPLE · ECE203 MIDTERM, FALL 2023, QUESTION 4 · CORE · OFFICIAL

Let $X \sim N(10, 4)$. Find a such that $P[aX \leq 1] = 0.15$, using nearest values in the Φ table.

$aX \sim N(a\mu, (a\sigma)^2)$ with $\mu = 10, \sigma = 2$. Assuming $a > 0$, $0.15 = \Phi(b)$ with $b = (1 - a\mu)/(a\sigma)$. The table gives $\Phi(1.04) \approx 0.85$, so $b = -1.04$, $1 = a(\mu - 1.04\sigma)$ and $a = 1/(10 - 1.04 \times 2)$, positive as assumed.

Joint distributions

Notes 22–29, Ross ch. 6. Summing or integrating the other variable away gives the **marginal**:

$$p_X(x) = \sum_j p_{XY}(x, y_j), \quad f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x, y) dy$$

X and Y are **independent** exactly when the joint factors into the two marginals ($F_{XY} = F_X F_Y$, $p_{XY} = p_X p_Y$, or $f_{XY} = f_X f_Y$); the support must be a rectangle as well as the formula splitting. For independent X, Y the density of $Z = X + Y$ is their **convolution**:

$$f_{X+Y}(z) = \int_{-\infty}^{\infty} f_X(z - y) f_Y(y) dy$$

A sum of independent normals is normal (means and variances add); a sum of independent Poissons is Poisson (rates add).

WORKED EXAMPLE · ECE203 TUTORIAL 8 (2021), EXAMPLE T8.2 · CORE · OFFICIAL

X and Y are independent rolls of a fair four-sided and a fair six-sided die. What is the pmf of $Z = X + Y$?

Discrete convolution: $p_Z(n) = \sum_k p_X(k) p_Y(n - k) = \frac{1}{4} \sum_{k=1}^4 p_Y(n - k)$, and each surviving term is $\frac{1}{24}$, so count how many of $n - 1, \dots, n - 4$ lie in $\{1, \dots, 6\}$: $p_Z(n) = \frac{1}{24}, \frac{2}{24}, \frac{3}{24}, \frac{4}{24}, \frac{4}{24}, \frac{4}{24}, \frac{3}{24}, \frac{2}{24}, \frac{1}{24}$ for $n = 2, \dots, 10$. A box convolved with a longer box is a trapezoid, flat from $n = 5$ to 7.

Expectation, generating functions and limit theorems

Notes 30–39, Ross ch. 7–8. Linearity of expectation holds for any pair, however entangled: $E[X + Y] = E[X] + E[Y]$. **Covariance** measures the gap that independence closes, and **correlation** normalises it into a fixed range ($\rho = 0$ does *not* imply independence):

$$\text{Cov}(X, Y) = E[XY] - E[X]E[Y], \quad \rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}[X] \text{Var}[Y]}} \in [-1, 1]$$

$$\text{Var}\left[\sum_{i=1}^n X_i\right] = \sum_{i=1}^n \text{Var}[X_i] + 2 \sum_{i < j} \text{Cov}(X_i, X_j), \quad E[X] = E[E[X | Y]]$$

The last is the **tower property**; the **MGF** of an independent sum is the product, $M_{X+Y}(t) = M_X(t) M_Y(t)$. **Chebyshev** gives the **weak law of large numbers**; the **central limit theorem** says how the sample mean is spread around μ , independent of what the X_i were:

$$P[|X - \mu| \geq b] \leq \frac{\sigma^2}{b^2}, \quad Z_n = \frac{X_1 + \cdots + X_n - n\mu}{\sigma\sqrt{n}} \longrightarrow N(0, 1)$$

WORKED EXAMPLE · ECE203 FINAL, FALL 2024, QUESTION 7 · CORE · OFFICIAL

One hundred independent $X_i \sim U(0, 1)$ are generated. You win if $X_1 + \cdots + X_{100} > 55$. Use the central limit theorem to approximate the probability that you win, expressed with $\Phi(\cdot)$.

For $U(0, 1)$, $E[X_i] = 0.5$ and $\text{Var}[X_i] = 1/12$. Centre the sum at $n\mu = 50$ and scale by $\sigma\sqrt{n} = \sqrt{100/12} = 10/\sqrt{12}$, so the event is $Z_n > 5/(10/\sqrt{12}) = \sqrt{3}$. By the CLT the answer is $1 - \Phi(\sqrt{3}) \approx 1 - \Phi(1.7321) \approx 0.0418$.

Practice, with answers

- (Tutorial 1, T1.3)** Show that $P[AB^cC^c] \geq P[A] - P[AB] - P[AC]$. *Answer:* split A into four disjoint pieces by B and C ; $ABC \cup AB^cC = AC$ and $ABC^c \subset AB$.
- (Midterm 2024, Q3)** Capacitors are bad with probability 0.1; a good one survives t seconds with probability e^{-at} , a bad one e^{-bt} , $b > a > 0$. What test length t_0 makes 99% of those sold good? *Answer:* Bayes gives $e^{(b-a)t_0} \geq 0.99/0.09 = 11$, so $t_0 \geq \ln 11/(b - a)$.
- (Midterm 2025, Q4)** $p_X(n) = 1/3^n$ for $n \geq 1$, $p_X(0) = p_0$. Find p_0 , $E[X]$ and $E[2^X]$. *Answer:* $p_0 = 1/2$, $E[X] = 3/4$, $E[2^X] = 2.5$ (not $2^{3/4} \approx 1.68$).
- (Tutorial 6, T6.3)** $X \sim N(0, 1)$ and $Z = \sqrt{|X|}$. Find f_Z . *Answer:* $F_Z(a) = 1 - 2\Phi(-a^2)$, so $f_Z(a) = \frac{2\sqrt{2}a}{\sqrt{\pi}} e^{-a^4/2}$ for $a \geq 0$, and 0 otherwise.
- (Final 2025, Q4)** $f_{XY} = x + y$ on $0 < x < a$, $0 < y < b$. When is it valid? With $a = b = 1$, find $P[X < Y]$ and $f_{X|Y}$. *Answer:* $ab(a+b) = 2$; $P[X < Y] = 1/2$; $f_{X|Y}(x | y) = (x+y)/(\frac{1}{2} + y)$, which depends on y , so not independent.
- (Final 2024, Q5)** Start with \$1; draw $N \sim \text{Poisson}(\lambda)$ and flip a fair coin N times, a head multiplying by 1.1, a tail by 0.9. Find $E[Y]$, $\text{Var}[Y]$. *Answer:* $E[Y] = 1$; $E[Y^2] = e^{\lambda/100}$, so $\text{Var}[Y] = e^{\lambda/100} - 1$.

Key points of the lesson (src/01_course_overview.jsx, run 74b04d), read 2026-09-25; problems are the course's own, as the lesson attributes them.